Horizon-Dependent Risk Aversion and the Timing and Pricing of Uncertainty

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Recent Successes in Asset Pricing theory

Asset pricing theory using long-run risk has been successful in...

- Equity premium
- Volatility puzzle
- Predictability
- Value premium

But puzzles remain concerning the interaction between the pricing and timing of risk:

- Preference for early versus late resolution of uncertainty?
- Short-term versus long-term risk prices

Why should we care?

- How to price new assets?
- Investment in the long-term versus short-term
- Very long-term investment (Climate change etc...)
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This paper

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**Horizon-dependent risk aversion (HDRA):** agents are more averse to short-horizon risk

Introducing this observed feature in a preference model with long-run risk can

- Match most standard asset pricing moments

- Explain remaining puzzles on:
  - the preference for early versus late resolution of uncertainty
  - the downward sloping term-structure of excess returns
Horizon-dependent risk aversion

- Risk aversion is ...
  - ... lower for distant risks
  - ... higher for imminent risks

![Diagram showing distant and imminent risk scenarios]

- Jones et al. (1973); Onculer (2000); Sagristano et al. (2002); Noussair et al. (2006); Coble et al. (2010); Baucells et al. (2010); Abdellaoui et al. (2011)
Results

Natural theory for downward sloping price of risk?
Natural theory for downward sloping price of risk?

Relation between horizon-dependent risk aversion and the term-structure of risk prices

- At first glance, relation seems very straightforward
- In a dynamic framework, things are not so simple
- Pseudo-recursive model with horizon-dependent risk aversion
  - Dynamic consistency for inter-temporal choices
  - Intra-temporal choices are time-inconsistent
Results

- **Time neutrality**
  - Agents are not time neutral
  - Preferences for late resolution of uncertainty AND a high price of risk can be compatible

- **Pricing impact of horizon-dependent risk aversion**
  - the pricing of immediate consumption shocks and drift shocks is unchanged from the standard model
  - the pricing of volatility shocks depends on the horizon-dependent risk aversion structure
  - downward sloping term structure for Sharpe ratios of equity excess returns
Some related literature

- Time premium puzzle: Epstein et al (2014)
- Empirical evidence for the term-structure of expected returns
  - Synthetic dividend strips in van Binsbergen et al. (2012) with 1.5-year maturity
  - Dividend futures contracts in van Binsbergen et al. (2015) with 1-7 year maturities across three world regions
  - Housing market in UK and Singapore for very long-term risk pricing in Giglio et al. (2014)
  - Using variance swaps in Ait-Sahalia et al. (2012), Dew-Becker et al. (2016)
- Production-based models with downward sloping term-structures of returns
  - Endogeneously decreasing risk in dividends in Ai et al. (2012), Croce et al. (2014)
  - Increasing contribution of negatively priced shocks in Kogan and Papanikolaou (2013)
- Preference-based rationalization
  - 1st order risk aversion models (Andries (2012), Curatola (2014))
Outline

1. Dynamic horizon-dependent risk aversion model
2. Early versus late resolutions of uncertainty
3. Pricing of risk
Plan

1. Dynamic horizon-dependent risk aversion model

2. Early versus late resolutions of uncertainty

3. Pricing of risk
HDRA and term-structure of risk prices: a straightforward relation?

- A very simple 3 period model with horizon-dependent risk aversion

\[ U_0 (\{C\}) = C_0 + \mathbb{E}_0 (C_1^{1-\gamma_1})^{\frac{1}{1-\gamma_1}} + \mathbb{E}_0 (C_2^{1-\gamma_2})^{\frac{1}{1-\gamma_2}} \]

with \( \gamma_1 > \gamma_2 \)
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with \( \gamma_1 > \gamma_2 \)

- Ratios of marginal utility:

\[
\frac{dU}{dC_1} \propto \left( \frac{C_1}{C_0} \right)^{-\gamma_1} \\
\frac{dU}{dC_2} \propto \left( \frac{C_2}{C_0} \right)^{-\gamma_2}
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- Ratios of marginal utility:

\[ \frac{dU}{dC_1} / \frac{dU}{dC_0} \propto \left( \frac{C_1}{C_0} \right)^{-\gamma_1} \]

\[ \frac{dU}{dC_2} / \frac{dU}{dC_0} \propto \left( \frac{C_2}{C_0} \right)^{-\gamma_2} \]

- An asset with payoff at time \( t = 2 \) will be priced with risk-aversion \( \gamma_2 \), and asset with payoff at time \( t = 1 \) will be priced with risk aversion \( \gamma_1 \)
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- An asset with payoff at time \(t = 2\) will be priced with risk-aversion \(\gamma_2\), and
  an asset with payoff at time \(t = 1\) will be priced with risk aversion \(\gamma_1\)

- Are we done?
HDRA and term-structure of risk prices: a straightforward relation?

- What happens if the agent can trade again at time $t = 1$?
HDRA and term-structure of risk prices: a straightforward relation?

- What happens if the agent can trade again at time $t = 1$?
- time inconsistent utility:

$$U_1 (\{C\}) = C_1 + \mathbb{E}_1 \left( C_2^{1-\gamma_1} \right)^{\frac{1}{1-\gamma_1}}$$
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$$U_1 (\{C\}) = C_1 + E_1 (C_2^{1-\gamma_1})^{\frac{1}{1-\gamma_1}}$$

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- At time $t = 1$, assets with payoff at time $t = 2$ are priced by risk aversion $\gamma_1$.
- At time $t = 0$, a sophisticated agent knows she will change her utility next period, and price $P_{1,2}$ with risk aversion $\gamma_1$. 
HDRA and term-structure of risk prices: a straightforward relation?

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- At time $t = 0$, she substitutes between current consumption and next period payoff $P_{1,2}$ with risk aversion $\gamma_1 \rightarrow$ the impact of $\gamma_2 < \gamma_1$ disappears!
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- At time $t = 0$, she substitutes between current consumption and next period payoff $P_{1,2}$ with risk aversion $\gamma_1 \rightarrow$ the impact of $\gamma_2 < \gamma_1$ disappears!
- Are we doomed?
A model with time-inconsistent intra-temporal risk aversion

Use Epstein-Zin preferences framework:

- Separate risk aversion and elasticity of intertemporal substitution
- Retain the effect of horizon dependent risk aversion in the valuation of wealth
- Build on the success of the long-run risk asset pricing literature
- Separate HDRA from other forms of time inconsistencies (hyperbolic discounting)
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- Start with:

\[
V_t = \left( (1 - \beta) C_t^{1-\rho} + \beta \mathbb{E}_t \left[ \tilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}
\]

with \( \gamma > 1, \rho > 0, \) AND
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with \( \gamma > 1, \ \rho > 0, \ AND \)

\[
\tilde{V}_{t+1} = \left( (1 - \beta) C_{t+1}^{1-\rho} + \beta \mathbb{E}_{t+1} \left[ \tilde{V}_{t+2}^{1-\tilde{\gamma}} \right] \right) \left( \frac{1}{1-\rho} \right)
\]

with \( \gamma > \tilde{\gamma} > 1 \)
A model with time-inconsistent intra-temporal risk aversion

- Inter-temporal decisions are dynamically consistent
- Intra-temporal decisions are time-inconsistent
- Assume the agent is sophisticated
- Assume the agent cannot commit (a representative agent assumption will be made)
Plan

1. Dynamic horizon-dependent risk aversion model

2. Early versus late resolutions of uncertainty

3. Pricing of risk
Assume an agent with risky consumption over time:

\[ c_{t+1} - c_t = \mu + \sigma W_{c,t+1} \]

- Value of the consumption stream is \( V \)
- Value if all shocks are revealed at \( t + 1 \) is \( V^* \)
- Term premium:

\[
TP = \frac{V^*_t - V_t}{V^*_t}
\]
Time neutrality

Time \( t \) distributions for all \( t + \tau \) risks are unchanged:

- Under expected utility \( V = V^* \) and \( TP = 0 \)

But...
Time neutrality

Time $t$ distributions for all $t + \tau$ risks are unchanged:

- Under expected utility $V = V^*$ and $TP = 0$

But...

- Under EZ preferences $V \neq V^*$ and $TP \neq 0$
- If $\gamma > \rho$, then $TP > 0$
- To explain the equity premium, we need $\gamma \approx 10$ and $\rho \approx 1$
- Epstein et al (2014): the term premium is above 30%!
HDRA and Time neutrality

Under HDRA:

$$TP = 1 - \exp \left( \frac{1}{2} \left( 1 - \gamma + (1 + \beta) (\gamma - \tilde{\gamma}) \right) \frac{\beta^2}{1 - \beta^2} \sigma^2 \right).$$

- $\gamma > \tilde{\gamma}$ so HDRA lowers the term premium
- Why? early resolution replaces long-horizon risk by short-horizon one
- If $\gamma < \rho + (1 + \beta) (\gamma - \tilde{\gamma})$ it becomes negative
- We can have $TP < 0$ AND $\gamma > \tilde{\gamma} > \rho$!
- High equity premium no longer imposes unrealistic preferences for early resolution
Risk Pricing and Time neutrality under HDRA

Timing premium

- Delayed risk aversion $\gamma$
- Fraction of consumption

Risk premium

- Delayed risk aversion $\gamma$
- Fraction of consumption

HDRA
EZ

2 4 6 8 10

HDRA
EZ
Plan

1 Dynamic horizon-dependent risk aversion model

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3 Pricing of risk
Pricing of risk in our model

The stochastic discount factor:

\[ \Pi_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \times \left( \frac{V_{t+1}}{E_t[V_{t+1}^{1-\gamma}]} \right)^{\rho-\gamma} \times \left( \frac{\tilde{V}_{t+1}}{V_{t+1}} \right)^{1-\gamma} \times \left( \frac{E_t[V_{t+1}^{1-\gamma}]}{E_t[\tilde{V}_{t+1}^{1-\gamma}]} \right)^{\rho-\gamma} \]

- I = the standard CRRA price for immediate risk
- II = EZ term for long-run shocks
- III = HDRA model: comes from dynamic inconsistency between \( V \) and \( \tilde{V} \)
Endowment economy

Lucas tree endowment economy with log consumption growth:

\[
 c_{t+1} - c_t = \mu + \phi c x_t + \alpha_c \sigma_t W_{t+1}
\]

\[
 x_{t+1} = \nu_x x_t + \alpha_x \sigma_t W_{t+1}
\]

\[
 \sigma_{t+1}^2 - \sigma^2 = \nu_\sigma \left( \sigma^2_t - \sigma^2 \right) + \alpha_\sigma \sigma_t W_{t+1}
\]

\(\nu_x\) contracting, \(\nu_\sigma < 1 - \frac{\alpha^2_\sigma}{2\sigma^2}\), and \(\alpha_c, \alpha_x, \alpha_\sigma\) orthogonal.
Closed-form solutions: $\rho = 1$

$$v_t - \tilde{v}_t = -\frac{1}{2} \beta (\gamma - \tilde{\gamma}) (\alpha^2_c + \phi^2 \alpha^2_x + (\psi(\tilde{\gamma}))^2 \alpha^2_\sigma) \sigma^2_t$$

where $\phi = \frac{\beta \phi_c}{1 - \beta \nu_x}$, and $\psi(\tilde{\gamma})$ is a function of the parameters of the model and term $\frac{1 - \beta \nu_\sigma}{\beta (1 - \gamma)}$.

- if volatility is constant, shocks affect only consumption levels (not its risk), which affects inter-temporal decision making $\rightarrow$ HDRA does not affect the pricing of risk
- volatility shocks affect intra-temporal decision making through time $\rightarrow$ HDRA impacts the pricing of such risk
- Closed-form solutions for the term-structure of risk-free and excess returns, and Sharpe ratios
## Calibration

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\Delta c)$</td>
<td>2%</td>
<td>1.8%</td>
</tr>
<tr>
<td>$\sigma(\Delta c)$</td>
<td>3%</td>
<td>3.2%</td>
</tr>
<tr>
<td>$AC1(\Delta c)$</td>
<td>0.29</td>
<td>0.20</td>
</tr>
<tr>
<td>$AC2(\Delta c)$</td>
<td>0.03</td>
<td>0.07</td>
</tr>
<tr>
<td>$AC3(\Delta c)$</td>
<td>−0.17</td>
<td>0.01</td>
</tr>
<tr>
<td>$E(\Delta d)$</td>
<td>1.3%</td>
<td>1.7%</td>
</tr>
<tr>
<td>$\sigma(\Delta d)$</td>
<td>11%</td>
<td>15%</td>
</tr>
<tr>
<td>$AC1(\Delta d)$</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>$\rho(\Delta c, \Delta d)$</td>
<td>0.52</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Data source: Shiller’s website, annual data 1926-2009
Impact on risk-free Bond returns

Evidence from van Binsbergen et al. (2015): 1-5y = 1.2%; 5-10y = 1.8%
Impact on Dividend Strips excess returns Sharpe ratios

Evidence from van Binsbergen et al. (2015): 1y = 0.12; 5y= 0.16; index= 0.04;
Impact on Variance Swaps returns Sharpe ratios

Evidence from Dew-Becker et al. (2016): 1m = -1.3; 3m= 0.07; 12m= 0.35;
Interpretation / implications

- Calibrated model matches standard asset pricing and macro moments
- HDRA’s impact on volatility risk pricing generates
  - downward sloping term-structure for dividend strips Sharpe ratios
  - upward sloping term-structure for variance swaps Sharpe ratios
- Consistent with empirical evidence
  - direct evidence from option data and variance swaps on the pricing of volatility risk
  - direct evidence from the dividend strips futures market
  - indirect evidence with the value premium
  - This simple version of HDRA cannot match the front-end of the curve evidence for variance swaps returns
- "Reasonable" ranges for preferences for early or late resolution of uncertainty
Conclusion

Start with two observations, strongly related at first glance
- empirical evidence for downward sloping expected returns in the term-structure
- micro/lab evidence for horizon-dependent risk aversion with low long-run risk aversion

Build on the success of the long-run risk literature to explain asset pricing moments

Preference-based approach with HDRA → address two puzzles on the pricing and timing of risk
- timing premium puzzle
- term-structure of the price of risk

A dynamic model with sophisticated agents shows
- risk prices are affected solely through the volatility shocks
- volatility risk pricing DOES generate a downward sloping term-structure for dividend strips risk pricing, an upward sloping term-structure for variance swaps risk pricing → success!

Further testable implications on liquid/illiquid assets, on dynamics of term-structure

Possible extensions
- expectation formation under time inconsistency
- pricing of the front-end of the term-structure