Horizon-Dependent Risk Aversion and the Timing and Pricing of Uncertainty

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Recent Successes in Asset Pricing theory

Asset pricing theory using long-run risk has been successful in...

- Equity premium
- Volatility puzzle
- Predictability
- Value premium

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- Preference for early versus late resolution of uncertainty?
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Why should we care?

- How to price new assets?
- Investment in the long-term versus short-term
- Very long-term investment (Climate change etc...)

This paper

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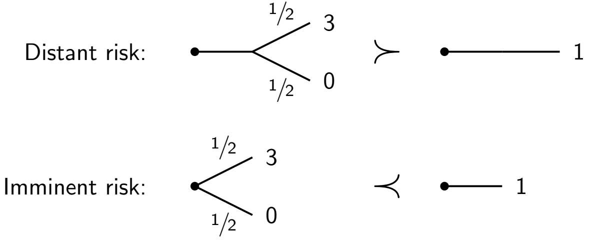
Introducing this observed feature in a preference model with long-run risk can

- Match most standard asset pricing moments
- Explain remaining puzzles on:
  - the preference for early versus late resolution of uncertainty
  - the downward sloping term-structure of excess returns

Horizon-dependent risk aversion

- Risk aversion is ...
  - ... **lower** for **distant** risks
  - ... **higher** for **imminent** risks



- Jones et al. (1973); Onculer (2000); Sagristano et al. (2002); Noussair et al. (2006); Coble et al. (2010); Baucells et al. (2010); Abdellaoui et al. (2011)

Results

Natural theory for downward sloping price of risk?

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Relation between horizon-dependent risk aversion and the term-structure of risk prices

- At first glance, relation seems very straightforward
- In a dynamic framework, things are not so simple
- Pseudo-recursive model with horizon-dependent risk aversion
    - Dynamic consistency for **inter**-temporal choices
    - **Intra**-temporal choices are time-inconsistent

Results

- Time neutrality
  - Agents are not time neutral
  - Preferences for late resolution of uncertainty AND a high price of risk can be compatible

- Pricing impact of horizon-dependent risk aversion
  - the pricing of immediate consumption shocks and drift shocks is unchanged from the standard model
  - the pricing of volatility shocks depends on the horizon-dependent risk aversion structure
  - downward sloping term structure for Sharpe ratios of equity excess returns

Some related literature

- Time premium puzzle: Epstein et al (2014)
- Empirical evidence for the term-structure of expected returns
  - Synthetic dividend strips in van Binsbergen et al. (2012) with 1.5-year maturity
  - Dividend futures contracts in van Binsbergen et al. (2015) with 1-7 year maturities across three world regions
  - Housing market in UK and Singapore for very long-term risk pricing in Giglio et al. (2014)
  - Using variance swaps in Ait-Sahalia et al. (2012), Dew-Becker et al. (2016)
  - Using index option straddles, Andries et al (2015)
- Production-based models with downward sloping term-structures of returns
  - Endogeneously decreasing risk in dividends in Ai et al. (2012), Croce et al. (2014)
  - Increasing contribution of negatively priced shocks in Kogan and Papanikolaou (2013)
- Preference-based rationalization
  - 1st order risk aversion models (Andries (2012), Curatola (2014))

Outline

1. Dynamic horizon-dependent risk aversion model
2. Early versus late resolutions of uncertainty
3. Pricing of risk

Plan

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HDRA and term-structure of risk prices: a straightforward relation?

- A very simple 3 period model with horizon-dependent risk aversion

$$U_0(\{C\}) = C_0 + \mathbb{E}_0\left(C_1^{1-\gamma_1}\right)^{\frac{1}{1-\gamma_1}} + \mathbb{E}_0\left(C_2^{1-\gamma_2}\right)^{\frac{1}{1-\gamma_2}}$$

with $\gamma_1 > \gamma_2$

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- Ratios of marginal utility:

$$\frac{dU/dC_1}{dU/dC_0} \propto \left(\frac{C_1}{C_0}\right)^{-\gamma_1}$$

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A model with time-inconsistent intra-temporal risk aversion

Use Epstein-Zin preferences framework:

- Separate risk aversion and elasticity of intertemporal substitution
- Retain the effect of horizon dependent risk aversion in the valuation of wealth
- Build on the success of the long-run risk asset pricing literature
- Separate HDRA from other forms of time inconsistencies (hyperbolic discounting)

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- Start with:

$$V_t = \left((1-\beta)\, C_t^{1-\rho} + \beta \mathbb{E}_t \left[\tilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}$$

with $\gamma > 1$, $\rho > 0$, *AND*

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$$\tilde{V}_{t+1} = \left((1-\beta)\, C_{t+1}^{1-\rho} + \beta \mathbb{E}_{t+1} \left[\tilde{V}_{t+2}^{1-\tilde{\gamma}} \right]^{\frac{1-\rho}{1-\tilde{\gamma}}} \right)^{\frac{1}{1-\rho}}$$

with $\gamma > \tilde{\gamma} > 1$

A model with time-inconsistent intra-temporal risk aversion

- Inter-temporal decisions are dynamically consistent
- Intra-temporal decisions are time-inconsistent
- Assume the agent is sophisticated
- Assume the agent cannot commit (a representative agent assumption will be made)

Plan

1. Dynamic horizon-dependent risk aversion model

2. Early versus late resolutions of uncertainty

3. Pricing of risk

Time neutrality

Assume an agent with risky consumption over time:

$$c_{t+1} - c_t = \mu + \sigma W_{c,t+1}$$

- Value of the consumption stream is V
- Value if all shocks are revealed at $t+1$ is V^*
- Term premium:

$$\text{TP} = \frac{V_t^* - V_t}{V_t^*}$$

Time neutrality

Time t distributions for all $t + \tau$ risks are unchanged:

- Under expected utility $V = V^*$ and $\mathrm{TP} = 0$

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- Under EZ preferences $V \neq V^*$ and $\mathrm{TP} \neq 0$
- If $\gamma > \rho$, then $\mathrm{TP} > 0$
- To explain the equity premium, we need $\gamma \approx 10$ and $\rho \approx 1$
- Epstein et al (2014): the term premium is above 30% !

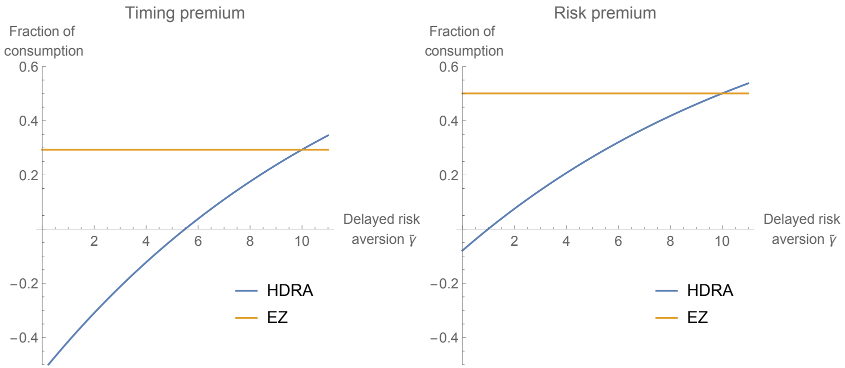
HDRA and Time neutrality

Under HDRA:

$$\mathrm{TP} = 1 - \exp\left(\frac{1}{2}\left(1 - \gamma + (1+\beta)(\gamma - \tilde{\gamma})\right)\frac{\beta^2}{1-\beta^2}\,\sigma^2\right).$$

- $\gamma > \tilde{\gamma}$ so HDRA lowers the term premium
- Why? early resolution replaces long-horizon risk by short-horizon one
- If $\gamma < \rho + (1+\beta)(\gamma - \tilde{\gamma})$ it becomes negative
- We can have $\mathrm{TP} < 0$ AND $\gamma > \tilde{\gamma} > \rho$!
- High equity premium no longer imposes unrealistic preferences for early resolution

Risk Pricing and Time neutrality under HDRA



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Pricing of risk in our model

The stochastic discount factor:

$$\Pi_{t,t+1} = \underbrace{\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}}_{\text{(I)}} \times \underbrace{\left(\frac{V_{t+1}}{E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}}_{\text{(II)}} \times \underbrace{\left(\frac{\tilde{V}_{t+1}}{V_{t+1}}\right)^{1-\gamma}\left(\frac{E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}{E_t\left[\tilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}}_{\text{(III)}}$$

- I $=$ the standard CRRA price for immediate risk
- II $=$ EZ term for long-rum shocks
- III $=$ HDRA model: comes from dynamic inconsistency between V and $\tilde{V}$

Endowment economy

Lucas tree endowment economy with log consumption growth:

$$c_{t+1} - c_t = \mu + \phi_c x_t + \alpha_c \sigma_t W_{t+1}$$

$$x_{t+1} = \nu_x x_t + \alpha_x \sigma_t W_{t+1}$$

$$\sigma_{t+1}^2 - \sigma^2 = \nu_\sigma \left(\sigma_t^2 - \sigma^2\right) + \alpha_\sigma \sigma_t W_{t+1}$$

ν_x contracting, $\nu_\sigma < 1 - \frac{\alpha_\sigma^2}{2\sigma^2}$, and α_c, α_x, α_σ orthogonal.

Closed-form solutions: $\rho = 1$

$$v_t - \tilde{v}_t = -\frac{1}{2}\beta(\gamma - \tilde{\gamma})\left(\alpha_c^2 + \phi^2\alpha_x^2 + (\psi(\tilde{\gamma}))^2\alpha_\sigma^2\right)\sigma_t^2$$

where $\phi = \frac{\beta\phi_c}{1-\beta\nu_x}$, and $\psi(\tilde{\gamma})$ is a function of the parameters of the model and term $\frac{1-\beta\nu_\sigma}{\beta(1-\tilde{\gamma})}$.

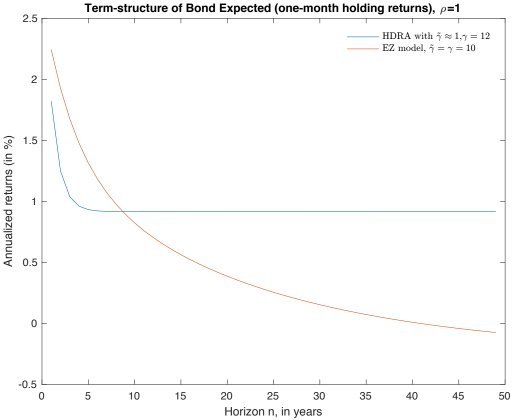
- if volatility is constant, shocks affect only consumption levels (not its risk), which affects inter-temporal decision making $\rightarrow$ HDRA does not affect the pricing of risk
- volatility shocks affect intra-temporal decision making through time $\rightarrow$ HDRA impacts the pricing of such risk
- Closed-form solutions for the term-structure of risk-free and excess returns, and Sharpe ratios

Calibration

Moment	Data	Model
$E(\Delta c)$	2%	1.8%
$\sigma(\Delta c)$	3%	3.2%
$AC1(\Delta c)$	0.29	0.20
$AC2(\Delta c)$	0.03	0.07
$AC3(\Delta c)$	-0.17	0.01
$E(\Delta d)$	1.3%	1.7%
$\sigma(\Delta d)$	11%	15%
$AC1(\Delta d)$	0.18	0.15
$\rho(\Delta c, \Delta d)$	0.52	0.56

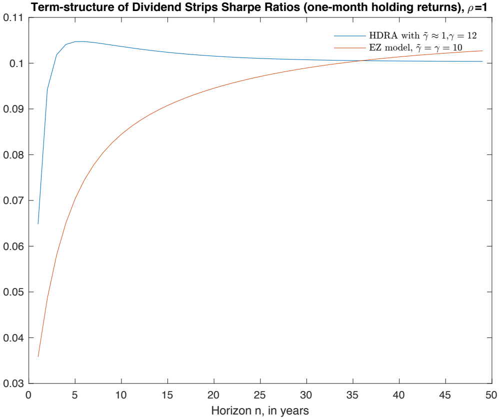
Data source: Shiller's website, annual data 1926-2009

Impact on risk-free Bond returns



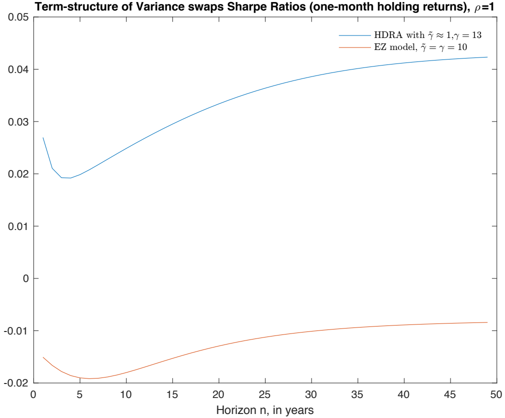
Evidence from van Binsbergen et al. (2015): 1-5y = 1.2%; 5-10y = 1.8%

Impact on Dividend Strips excess returns Sharpe ratios



Evidence from van Binsbergen et al. (2015): 1y = 0.12; 5y= 0.16; index= 0.04;

Impact on Variance Swaps returns Sharpe ratios



Evidence from Dew-Becker et al. (2016): 1m = -1.3; 3m= 0.07; 12m= 0.35;

Interpretation / implications

- Calibrated model matches standard asset pricing and macro moments
- HDRA's impact on volatility risk pricing generates
  - downward sloping term-structure for dividend strips Sharpe ratios
  - upward sloping term-structure for variance swaps Sharpe ratios
- Consistent with empirical evidence
  - direct evidence from option data and variance swaps on the pricing of volatility risk
  - direct evidence from the dividend strips futures market
  - indirect evidence with the value premium
  - This simple version of HDRA cannot match the front-end of the curve evidence for variance swaps returns
- "Reasonable" ranges for preferences for early or late resolution of uncertainty

Conclusion

- Start with two observations, strongly related at first glance
  - empirical evidence for downward sloping expected returns in the term-structure
  - micro/lab evidence for horizon-dependent risk aversion with low long-run risk aversion
- Build on the success of the long-run risk literature to explain asset pricing moments
- Preference-based approach with HDRA $\rightarrow$ address two puzzles on the pricing and timing of risk
  - timing premium puzzle
  - term-structure of the price of risk
- A dynamic model with sophisticated agents shows
  - risk prices are affected solely through the volatility shocks
  - volatility risk pricing DOES generate a downward sloping term-structure for dividend strips risk pricing, an upward sloping term-structure for variance swaps risk pricing $\rightarrow$ success!
- Further testable implications on liquid/illiquid assets, on dynamics of term-structure
- Possible extensions
  - expectation formation under time inconsistency
  - pricing of the front-end of the term-structure