Discounting and risk adjusting non-marginal investment projects

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January 28, 2011

Abstract

Standard cost-benefit analyses and asset pricing theories are based on the assumption that investment projects have marginal impacts on the consumption flows of stakeholders, so that social values and prices are not affected. This may not be true for large projects, such as those related to climate change or to the implementation of infrastructure projects in developing countries. In this paper, we explore qualitatively and quantitatively the error that is made when using the standard evaluation methods for non marginal projects. In particular, we discuss the importance of adapting the discount rate and the risk premium to the size of the investment projects under consideration.

Keywords: Cost-benefit analysis, discounting, risk premium, asset pricing.

Standard Cost-Benefit analyses use the classical marginalist approach to value investments and assets. Under this approach, prices and values express marginal rates of intertemporal substitution. For example, in the recent debate on the discount rate (Arrow (1999), Stern (2007), Weitzman (2007), Gollier (2008), Gollier and Weitzman (2010)), the efficient discount rate is derived by considering a marginal transfer of consumption through time. Thus, this discount rate can formally be used to evaluate an investment project only if its impact on the consumption path of the representative agent is small. In a similar fashion, the way we take account of risk when evaluating a project is usually made by assuming that the project does not change the aggregate risk. This is exactly what we do when we use the CAPM formula to price risk. This approach makes sense to express prices that sustain equilibrium with divisible goods, but this requires knowing the allocation at equilibrium. This approach also makes sense when one normatively evaluates a marginal action along the current equilibrium consumption path. It does not make sense when one evaluates non-marginal projects. Non marginal projects are those which have an impact on the structure of the stochastic consumption path, so that they affect equilibrium prices and normative values. Discount rates and risk premiums become endogenous in that case.

Let us illustrate this point with two examples. The first example is in the context of climate change. In Diez, Hope and Patmore (2007), the expected damages due to climate change in the business-as-usual "high-climate" scenario is evaluated to 13.8% of world GWP in 2200. The 5–95% confidence interval spans a range from 2.9% to 35.2% of GWP. Consider a strategy that would eliminate these damages at some non-marginal cost. If we use the classical approach of discounting, should we use the extended Ramsey rule with a reduced growth rate to take into account of the increasing damages, and with an increased uncertainty on growth coming from the uncertainty about these damages? As claimed by Stern (2007), this is problematic if the aim of the policy is precisely to reduce the intensity and the uncertainty of climate change!

The second example is provided by Diez and Cameron (2010), and is about a large infrastructure project in Laos. The Nam Theun II hydropower dam project has a

generation capacity of 1 Giga Watt from a 350 meters difference in elevation between the reservoir and the power station. The construction cost was US$ 1.3 billion, to be compared to gross consumption of the country which is around US$ 2.5 billion. The construction started in 2005, and was completed in the spring of 2010. The export of the electricity is expected to yield an annual benefit of US$ 250 million. From these figures, it is clear that the implementation of the project does affect the growth rate of the economy, and the willingness to invest for the future. Therefore, the choice of the discount rate to evaluate the project and to optimize its size must be endogenously determined.

This second example raises an important problem, which is the reference economy whose size should be compared to the size of the project to determine whether the latter is marginal or non-marginal. In fully integrated capital markets, i.e., in a globalized economy, all costs and all benefits will be efficiently disseminated and shared by all consumers of the planet. In this context, the relevant model has a representative agent who consumes the world GDP per capita and who bears and receives its fair share of the costs and benefits of the project. Thus, the marginal nature of the project would be measured in this context by relating the global costs and benefits of the project to the world GDP. But in reality, costs and benefits are not efficiently disseminated in the world economy, and risks are not efficiently shared among the 6 billion inhabitants of the earth. This implies that the existence of a representative agent is a fiction. In the case of the Nam Theun II hydropower dam project, it is strongly believed that most of the impacts of the dam will be borne by the Laos population, whose consumption will be non-marginally affected by the project.

When comparing different non-marginal policies, one needs to go back to the basic principles of public economics. If option A yields an uncertain consumption path $\left\{c_t^A\right\}_{t=0,1,\ldots}$ and if option B yields an uncertain consumption path $\left\{c_t^B\right\}_{t=0,1,\ldots}$, option A dominates option B if and only if it yields a larger discounted expected utility:

$$\sum_{t=0} e^{-\delta t} Eu\left(c_t^A\right) \geq \sum_{t=0} e^{-\delta t} Eu\left(c_t^B\right), \tag{1}$$

where δ is the social rate of pure preference for the present, and u is the increasing and concave utility function of the representative agent. This approach is rarely used in cost-benefit analyses, probably because of the complexity of the problem. Indeed, it requires a full description of the utility function, of the rate of pure preference for the present, and of the joint probability distribution of the status-quo consumption and of the payoff of the action. In spite of these challenges, this approach to the evaluation of non-marginal projects was undertaken by Nordhaus and Boyer (2000), Stern (2007), and Nordhaus (2008). Tol (2005), who reviewed the empirical literature on the estimation of the shadow value of emission abatement, showed that 62 of the 103 estimations of shadow value of carbon ignored the non-marginal nature of the impacts of climate change and of our global strategy to limit them.

Following Diez and Hepburn (2010), we hereafter examine the error that results from following the classical discounting approach when evaluating non-marginal projects.

1. *The discount rate*

Suppose that we use the classical discounting approach to evaluate a project that has a non-marginal impact on the growth of consumption. What is the sign and the size of the error that one does on the true value of the project? Concerning the sign of the effect, the intuition is quite simple. If the project is standard, with a cost incurred today for a sure benefit in the future, investing in the project will raise the expected growth rate of consumption. It will increase the discount rate through the wealth effect. Thus, the classical discounting approach will rely on a too small discount rate. Therefore, if it underestimates the discount rate, it overestimates the social value of the project.

To examine this question, consider a project that reduces current consumption by k today, and that increases consumption by a sure amount x at some specific date t. What is the maximum cost k that one is ready to incur today to get x at date t? In other words, what is the present value of increasing consumption by x at date t? The maximum cost that one is

ready to incur today to get x at date t is a function $k_t(x)$ whose properties are explored in this section. This function is defined as follows:

$$u(c_0 - k_t(x)) + e^{-\delta t} Eu(c_t + x) = u(c_0) + e^{-\delta t} Eu(c_t), \tag{2}$$

where c_0 and c_t are consumption levels in the status-quo scenario respectively at dates 0 and t. If the maximum cost is incurred, investing has no effect on the intertemporal utility of the agent. This means that $k_t(x)$ is the current social value of x consumed at date t. It is useful to define a size-adjusted discount rate $r_t(x)$ such that $k_t(x) = xe^{-r_t(x)t}$.

1.1. The marginal discount rate

Of course, we see from (2) that $k_t(0) = 0$. But what do we know about $k_t'(0)$? Differentiating equation (2) with respect to x yields

$$k_t'(x) = \frac{e^{-\delta t} Eu'(c_t + x)}{u'(c_0 - k_t(x))}, \tag{3}$$

which is positive. Because $k_t'(0) = e^{-r_t(0)t}$, it yields

$$r_t(0) = \delta - \frac{1}{t} \ln \frac{Eu'(c_t)}{u'(c_0)}. \tag{4}$$

This is the standard (marginal) discount rate used in the literature, for example by Arrow (1999), Weitzman (2007), or Gollier (2008). Ramsey (1928) used a specification with constant relative risk aversion γ ($u'(c) = c^{-\gamma}$) and a constant growth rate μ of consumption ($c_t = c_0 e^{gt}$), so that equation (4) simplifies to

$$r_t(0) = \delta + \gamma \mu. \tag{5}$$

This is the Ramsey rule, which states that the efficient discount rate net of the rate of pure preference for the present equals the product of relative risk aversion by the growth rate of consumption. Because marginal utility is decreasing, the impact of increasing consumption on welfare is decreasing with time in a growing economy. This wealth effect raises the discount rate above the rate of pure preference for the present. The larger the growth rate, the larger the discount rate.

1.2. Estimation of the error for non-marginal projects

The above result just states that the linear extrapolation $k_t(x) \simeq xe^{-r_i(0)t}$ is exact for marginal projects, i.e., a project in which x tends to zero. In this section, we estimate the error that one makes when using this standard evaluation approximation for non-marginal projects. Differentiating once again equation (3) implies that

$$k_t''(x) = \frac{k_t'(x)^2 u''(c_0 - k_t) + e^{-\delta t} Eu''(c_t + x)}{u'(c_0 - k_t)}. \tag{6}$$

Because u is concave, this is unambiguously negative. Thus, the valuation function $k_t(x)$ is increasing and concave. It implies that the linear extrapolation formula $k_t(x) = xe^{-r_i(0)t}$ which is systematically used in cost-benefit analyses overestimates the true social value of all projects with positive future cash flows.

One can estimate the order of magnitude of the valuation error by considering the following numerical example. Normalize current consumption to unity. Suppose that the growth rate of consumption is a safe 2%, that relative risk aversion is a constant equalling 2, and that the rate of impatience is zero. In this framework, the marginal discount rate is 4%. The true present valuation function $k_t(x)$ is depicted in Figure 1 for a project with a 1-year time horizon ($t=1$). It appears that it is very quickly diverge from $xe^{-0.04}$. For example, for a benefit that represents 10% of current consumption, the true present value is $k_t(0.1) = 8\%$, which should be compared to the traditional valuation $0.1e^{-0.04} = 9.6\%$. The (over-)estimation error represents one fifth of the true present value.
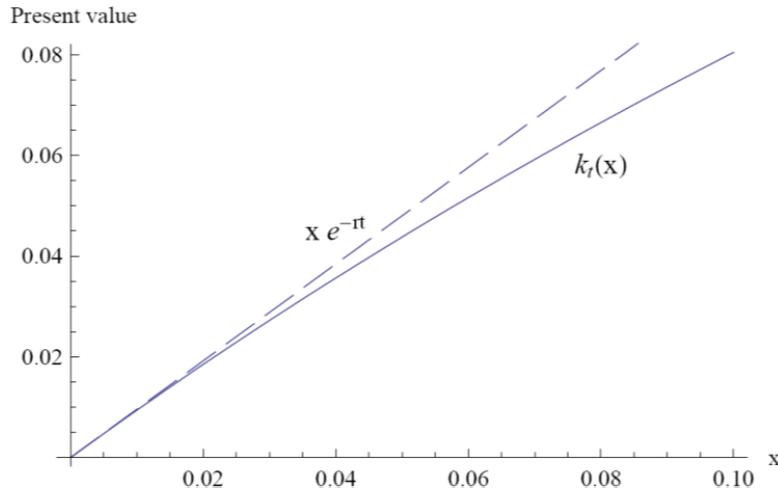
Figure 1: The true present valuation function as a function of the size x of the future benefit. We assume that $t=1$, $c_0 = 1$, $c_1 = 1.02$, $\delta = 0$, and $u'(c) = c^{-2}$. The dashed line corresponds to the present value extrapolated from the Ramsey rule ($r = 4\%$).

1.3. The size-adjusted discount rate

The use of an explicit welfare function to evaluate non-marginal projects may be cumbersome for practionners. We hereafter elaborate an alternative approach in which we preserve the basic discounting approach, but in which we adapt the discount rate to take into account the size of the project. This may be done by defining the size-adjusted discount rate $r_t(x)$ that we implicitly defined by $k_t(x) = xe^{-r_t(x)t}$, where $k_t(x)$ is defined by condition (2). If the cost of the project is less (larger) than its present value defined by $k_t(x)$, its implementation will obviously raise (reduce) the intertemporal welfare, so that $r_t(x)$ can indeed be interpreted as a size-adjusted discount rate. It can be rewritten explicitly as

$$r_t(x) = -\frac{1}{t}\ln\frac{k_t(x)}{x}. \tag{7}$$

Using the L'Hospital's rule, we obtain the standard formula for marginal projects:

$$r_t(0) = -\frac{1}{t}\ln k'(0) = \delta - \frac{1}{t}\ln\frac{Eu'(c_t)}{u'(c_0)}, \tag{8}$$

where the second equality is obtained from (3). We are interested in measuring the sensitivity of the discount rate in the neighborhood of small benefits. By equation (7), we have that

$$r_t'(x) = -\frac{1}{t}\frac{k_t'(x)x - k_t(x)}{xk_t(x)}. \tag{9}$$

Using L'Hospital's rule twice, we obtain :

$$r_t'(0) = -\frac{1}{t}\lim_{x\to 0}\frac{k_t''(x)x}{k_t(x)+xk_t'(x)} = -\frac{1}{t}\lim_{x\to 0}\frac{k_t'''(x)x+k_t''(x)}{2k_t'(x)+xk_t''(x)} = -\frac{k_t''(0)}{2tk_t'(0)}. \tag{10}$$

From equations (4) and (6), we have that

$$
\begin{aligned}
-\frac{k_t''(0)}{k_t'(0)} &= -\frac{k_t'(0)^2 u''(c_0) + e^{-\delta t}Eu''(c_t)}{u'(c_0)k_t'(0)} \\
&= k_t'(0)\left(-\frac{u''(c_0)}{u'(c_0)}\right) + \frac{e^{-\delta t}Eu'(c_t)}{u'(c_0)k_t'(0)}\left(-\frac{Eu''(c_t)}{Eu'(c_t)}\right) \\
&= e^{-r_t(0)t}\frac{R_0}{c_0} + \frac{R_t}{Ec_t}.
\end{aligned}
\tag{11}
$$

where $R_0 = -c_0 u''(c_0)/u'(c_0)$ is the index of relative risk aversion evaluated at c_0, and $R_t = -Ec_t Eu''(c_t)/Eu'(c_t)$ is the risk-adjusted relative risk aversion at date t. Combining equations (10) and (11) yields

$$r_t'(0)Ec_t = \frac{e^{(\mu_t - r_t(0))t}R_0 + R_t}{2t}, \tag{12}$$

where $e^{\mu_t t} = Ec_t/c_0$ is the annualized growth rate of expected consumption between dates 0 and t. Notice that the left-hand side of the above equation is the quasi-elasticity of the discount rate relative to the size of the cash-flow in the neighborhood of $x=0$. It measures the percentage increase in the efficient discount rate when the cash-flow at date t increases by 1% of expected consumption. When t is normalized to unity, the right-hand side of this equality is close to the average of relative risk aversion evaluated at dates 0 and t.

Let us reconsider the numerical example of the previous section, with $t=1$, $c_0 = 1$, $c_1 = 1.02$, $\delta = 0$, and $u'(c) = c^{-2}$. It yields $R_0 = R_1 = 2$ and $Exp(\mu_1 - r_1(0)) = 0.98$. Consider a benefit that represents 1% of consumption at date 1. Adjusting for the size of

this benefit would require increasing the discount rate from *4%* to $4\% + 1\% \times (0.98 \times 2 + 2)/2 = 5.98\%$. In Figure 2, we draw function $r_t(x)$ for benefits x up to 10% of future GDP. The dashed line corresponds to size-adjusted rate from the first-order Taylor approximation $r_t(x) \simeq r_t(0) + r_t^{'}(0)x$.
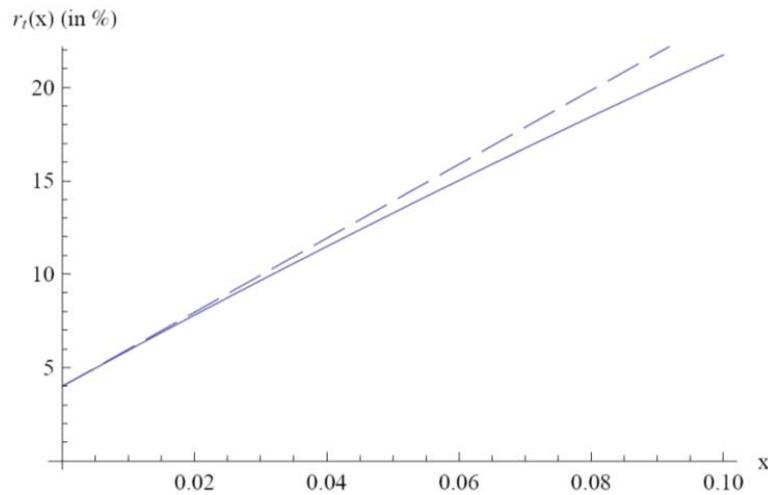

Figure 2: The size-adjusted discount rate as a function of the size x of the future benefit. We assume that $t=1$, $c_0 = 1, c_1 = 1.02$, $\delta = 0$, and $u'(c) = c^{-2}$.

2. *The risk premium*

Standard asset pricing formulas from the classical theory of finance are also valid only for marginal risks. Let us for example re-examine the theorem of Arrow and Lind (1970) that states that the risk premium should be zero if the cash-flows are risky but independent of the risk on aggregate consumption. This result is justified by the observation that risk aversion is of the second order on the certainty equivalent. When the size of risk tends to zero, its risk premium tends to zero as the square of the size of the risk. Consider a risky cash-flow $\mu + xy$ at date t, where y is a zero-mean risk, x is a scalar that characterizes the size of the risk on the cash-flow, and μ is the expected cash-flow.

Let us consider the compensating risk premium $\pi_c(x)$ which is implicitly defined by the following equality:

$$Eu(c_t + \mu + xy + \pi_c(x)) = Eu(c_t + \mu). \tag{13}$$

The compensating risk premium is the amount to pay to the risk bearer to compensate her for the risk. In general, it differs from the standard risk premium, which is the equivalent sure reduction in consumption that has the same effect on expected utility than the risk under consideration. But for small risks, the classical risk premium and the compensated risk premium are equal.

Of course, $\pi_c(0) = 0$. Differentiating equation (13) with respect to x yields

$$E(y + \pi_c'(x))u'(c_t + \mu + xy + \pi_c(x)) = 0. \tag{14}$$

It implies that

$$\pi_c'(x) = -\frac{Ey\, u'(c_t + \mu + xy + \pi_c(x))}{Eu'(c_t + \mu + xy + \pi_c(x))}. \tag{15}$$

The right-hand side of this equality is non-negative, since y and u' are negatively correlated when x is positive. By the covariance rule, it implies that $Eyu' \leq EyEu' = 0$. However, when x tends to zero, we have that

$$\pi_c'(0) = -\frac{Ey\, u'(c_t + \mu)}{u'(c_t + \mu)} = -Ey = 0. \tag{16}$$

This is the Arrow-Lind theorem. Marginal risks that are uncorrelated to the economy have no social cost. But what can we say about non-marginal independent risks? Differentiating equation (14) again implies that

$$E(y + \pi_c'(x))^2 u''(c_t + \mu + xy + \pi_c(x)) = -\pi_c''(x)\, Eu'(c_t + \mu + xy + \pi_c(x)). \tag{17}$$

Observe that the left-hand side of this equality is uniformly negative under risk aversion. It implies that the compensating risk premium is an increasing and convex function of the size of risk. This result does not hold for the classical risk premium, as shown by a counter-example presented in Eeckhoudt and Gollier (2001).

One can evaluate the error when estimating the risk premium by using the Arrow-Lind theorem. Using equation (17) around $x=0$ and assuming $\mu=0$ for the sake of a simple notation, we obtain that

$$\pi_c^{''}(0) = -\frac{Ey^2 Eu''(c_t)}{Eu'(c_t)} = \frac{Ey^2 R_t}{Ec_t},\tag{18}$$

where $R_t = -Ec_t Eu''(c_t)/Eu'(c_t)$ is the risk-adjusted relative degree of risk aversion at date t. The second order Taylor approximation of the compensated risk premium around $x=0$ implies that

$$\frac{\pi_c(x)}{Ec_t} \simeq 0.5\, Var\left(\frac{xy}{Ec_t}\right) R_2,\tag{19}$$

which is the Arrow-Pratt approximation. This means that the risk premium expressed as a percentage of initial expected consumption is approximately equal to half times the product of the variance of the relative change in consumption by the risk-adjusted relative risk aversion. For example, if the standard deviation of the cash-flow of the project equals 5% of aggregate consumption and relative risk aversion equals 2, the risk premium is approximately equal to one-fourth of a percent of aggregate consumption. This approximation is exact when y is log normally distributed, c_t is constant, and the utility function belongs to the CRRA family.

3. Conclusion

The beauty and usefulness of cost-benefit analysis is that it relies on a few numbers, which represent the social value of the different dimensions of costs and benefits: the value of life, the value of environmental assets, the discount rate, or the risk premium for example. Once these values are determined, the evaluator is just required to estimate the flows of these multi-dimensional impacts, and to value them according to these prices. We have shown in this paper that this simple toolbox can be used only if the actions under scrutiny are marginal, i.e., if implementing them has no macroeconomic effects. Otherwise, one needs to go back to the basics of public economics to evaluate these actions. Alternative

non-marginal strategies need to be compared through their impact on the social welfare function, whose description may raise new questions and new challenges in the public debate. For example, the problem of whether one should fully decarbonize our economies is a crucial policy question. With Stern (2007), I claim that this question cannot be answered via the standard cost-benefit tools just because decarbonizing our economies will radically transform our economic production processes, the structure of consumption, or the quality of the environment, for example. This will in turn affect the price vectors, as the discount rate, the risk premium, and the value of environmental assets.

Going back to the basic tools of public economics does generate new challenges that are particularly crucial in the case of non-marginal investments. An important difficulty is about the disentanglement of the attitude towards risk, time and inequalities, as examined for example by Zuber and Asheim (2010). Moreover, one should also recognize that welfare is a complex function of several attributes: consumption, social cohesion, quality of the environment, and so on. The role of consumption habit formation should also be clarified when the impacts of our action are non-marginal.

References

Arrow, K. J. (1999). Discounting and Intergenerational Equity. In Portney and Weyant (eds), *Resources for the Future*, Washington DC.

Arrow, K.J. and Lind, R.C. (1970). Uncertainty and the evaluation of public investment decision. *American Economic Review* 60: 364-378.

Diez S., Hope, C. and Patmore, N. (2007). Some economics of "dangerous" climate change: Reflections on the Stern Review. *Global Environmental Change* 17: 311-325.

Diez, S., and Hepburn, C. (2010). On non-marginal cost-benefit analysis. Grantham Research Institute on Climate Change and the Environment, WP18.

Eeckhoudt, L., and Gollier, C. (2001). Which shape for the cost curve of risk? *Journal of Risk and Insurance* 68: 387-402.

Gollier, C. (2008). Discounting with fat-tailed economic growth. *Journal of Risk and Uncertainty* 37: 171-186.

Gollier, C., and Weitzman, M.L. (2010). How Should the Distant Future be Discounted When Discount Rates are Uncertain? *Economic Letters* 145: 812-829.

Nordhaus, W.D. (2008). *A Question of Balance: Weighing the Options on Global Warming Policies*. New Haven: Yale University Press.

Nordhaus, W.D., and Boyer, J. (2000). *Warming the World: Economic Models of Global Warming*. Cambridge, MA: MIT Press.

Ramsey, F.P. (1928). A mathematical theory of savings. *The Economic Journal* 38: 543-59.

Stern, N. (2007). *Stern Review: The Economics of Climate Change*. Cambridge, UK: Cambridge University Press.

Tol, R.S.J. (2005). The marginal damage costs of carbon dioxide emissions: an assessment of the uncertainties. *Energy Policy* 33: 2064-2074.

Weitzman, M. L. (2007). Subjective expectations and asset-return puzzle. *American Economic Review* 97: 1102-1130.

Zuber, S. and Asheim, G.B. (2010). Justifying social discounting: the rank-discounted utilitarian approach. CESifo Working Paper 3192.