

# The role of relative prices in discounting

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IDEI Conference  
Toulouse  
May 09

# Stern Review

- Climate Change the biggest externality in human history.
- 5-20% of future GDP
- Enormous uncertainties in calculation:
- Feedback from cloudformation
- Feedback from methane release
- Feedback from ice-melting (Albedo)
- *Guess which is biggest uncertainty?*

# Stern Review

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- 5-20% of future GDP
- Enormous uncertainties in calculation:
- Feedback from cloudformation
- Feedback from methan release
- Feedback from ice-melting (Albedo)
- **DISCOUNT RATE!**

**Discounting and relative prices in  
future environmental damages**

**Michael Hoel & Thomas Sterner**  
*Climatic Change*

**An even Sterner Review**

Thomas Sterner Martin Persson  
*Review of Env Economics & Policy*

# Conventional Discounting

- If some cost or benefit component at a future date  $t$  is of the magnitude  $V_t$  and the discount rate is  $r$ , the present value is

- 

$$(1 + r)^{-t} V_t$$

# The effect is **big**

- 1 billion in 400 years = 3 \$ today (5%).
- in 500 years would be 2 cents.
- With 6% would be .02 cents.
- Difference between 5 / 6 % is a factor 100

# PROBLEM ?

- 1\$ in bank today = 2\$ in 6 years
- so \$2 cost in 6 years  $\sim$  cost of \$1 today
  
- How big in 240 years?
  
- Can economy grow one million million\*?

# Many Issues

- Can growth continue forever?
- Psychological aspects
- Hyperbolic and Gamma Discounting
- Risk
- Other considerations in U
- **RELATIVE PRICES**



## Correct value of future project

- $V_t = V_0(1+r)^{-t} (1+p)^t$
- The effect of relative prices can be as big as discounting!
- If  $p$  is big enough?

# Labour

- 100 years ago 10% of the population in Toulouse had a maid.
- Incomes are growing 5%/year

# Labour

- 100 years ago 10% of the population in Toulouse had a maid.
- Incomes grow 3-5%/year
- **How many** people have a maid today?

Why can't we all have maids?

# Why can't we all have maids?

- $P_{\text{maid}} = f(\text{Income})$

# FOOD

- World Agriculture is 24% GDP
- Assume we loose 1% of World Agriculture. How big is loss?
- Roughly  $0.01 * 0.24 = 0.24\%$  GDP

# FOOD

- World Agriculture is 24% GDP
- Assume we loose 95% of World Agriculture. How big is loss?
- Roughly  $0.95 * 0.24 = 23\%$  GDP

**-95% FOOD**

**• -23%?**



What is wrong?

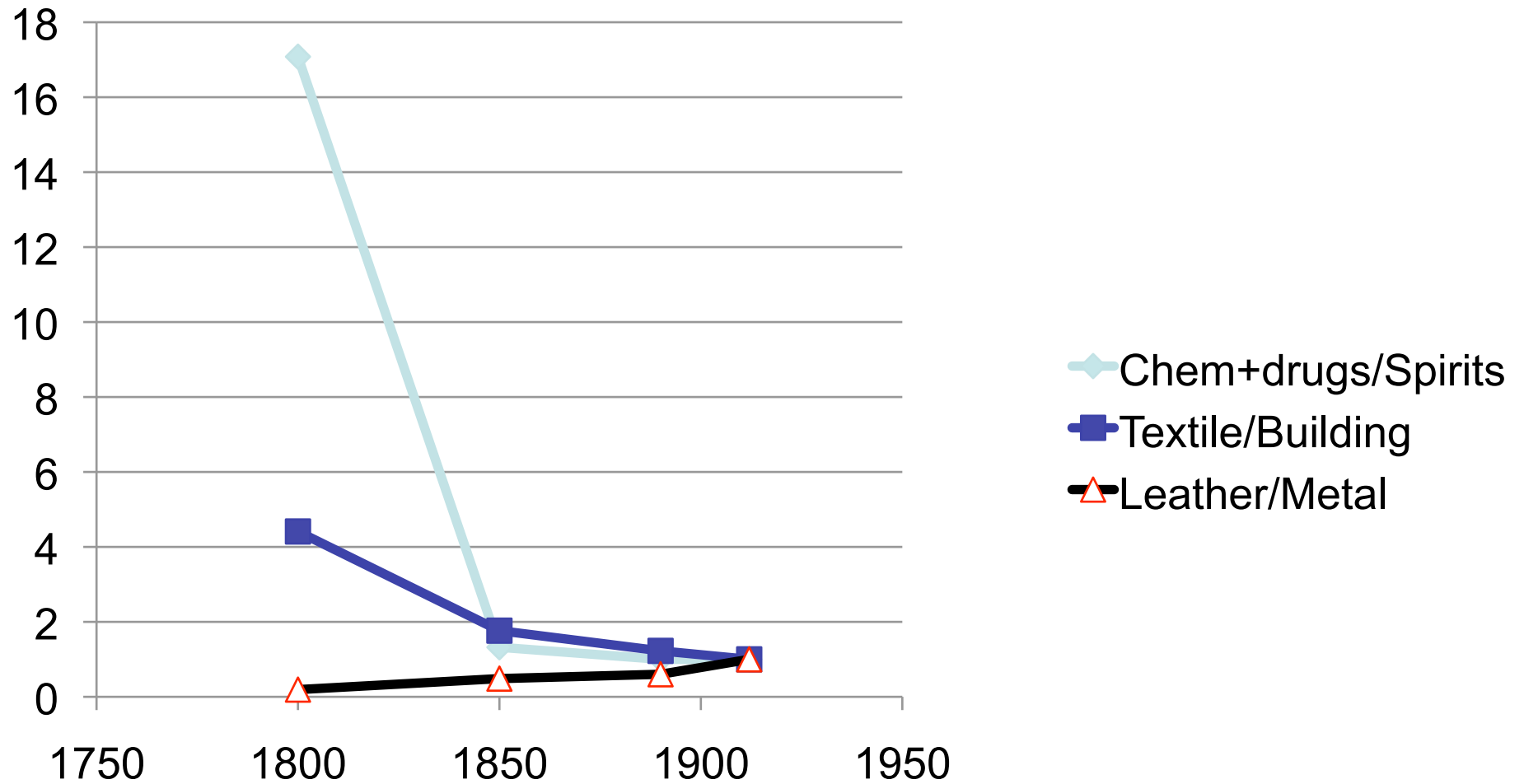
# What is wrong?

- Relative Prices of food...

# What is wrong?

- Relative Prices of food...
- will change **so fast**
- That the 95% loss will be worth  
>> 23% GDP.

# Relative prices, US fed gov 1800-1921



# Relative prices

- The price of "spirits" relative to "chemicals" rose 1700 %
- Similar for metals.
- $P(\text{flour}) / P(\text{wheat})$  falls
- Or "nails" in relation to "iron"
- $P(\text{labour})$ , results of mechanisation

# Future Ecosystem Scarcities

- Water
- Soil
- Wild (non-cultivated) fish
- Biodiversity
- Glaciers and snow
- Wildlife, protected areas
- Fuelwood, pasture, silence (?)

# OK: Why discount?

- We are impatient
- We will be richer
- Rich people dont know the value of money

$$r(t) = \rho + \alpha g_c(t)$$

Assume an intertemporal welfare function

$$W = \int_0^T e^{-\rho t} U(C(t)) dt$$

The tradeoffs between consumption at different points of time are given partly by the “utility discount rate”  $\rho$  partly by the utility function  $U$ .



The appropriate discount rate is the sum of these two reasons

$$r = \rho - \frac{\frac{d}{dt} U'(C(t))}{U'(C(t))}$$

With Constant elasticity of utility function  $\rightarrow$  classical Ramsey Rule

$$U(C) = \frac{1}{1-\alpha} C^{1-\alpha}$$

$$r(t) = \rho + \alpha g_C(t)$$

# Ramsey and growth

- If  $\rho = 0.01$ ,  $\alpha = 1.5$  and  $g = 2.5\%$   $r = 4.75\%$ .
- Constant iff growth is constant.
- Increases with growth
- If growth falls, future discount rates will fall over time. Azar & Sterner (1996): limits to growth  $\rightarrow$  falling discount rates and higher damage from carbon emissions.

# Compare Nordhaus 5 \$/ton

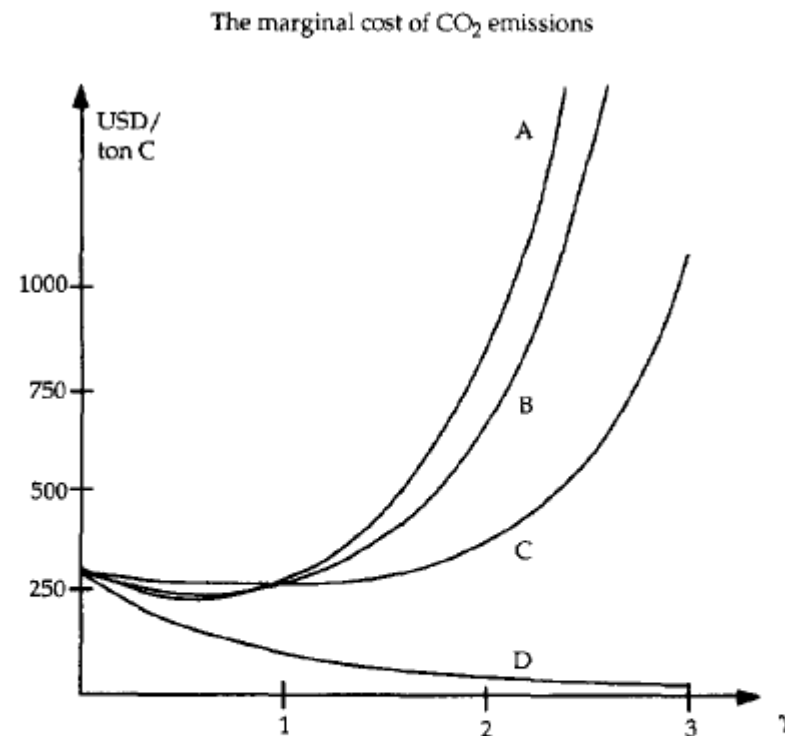


Fig. 3. The *generalized* cost of a unit emission of CO<sub>2</sub> is plotted as a function of  $\gamma$  in four cases. In plot A, B and C, the inequality situation is worsened, unchanged, and improved, respectively. In plot D, income distribution is not considered. The higher the value for  $\gamma$ , the higher is the discount rate, but also the inequality aversion.

# Results

- Nordhaus 5
- We got 10 –150 for gamma 3→0,5
- Falls with gamma !
- But--
- With inequality we got 250-1000
- Higher values for higher gamma!
- However we did assume  $Y_{\max} = 10Y$

# Are there Limits to Growth?

- Clearly YES:
- A finite planet
- The amount of cement, carbon, steel and water that we can use is limited!

# Are there Limits to Growth?

- Clearly YES:
- A finite planet
- The amount of cement, carbon, steel and water that we can use is limited!
- Clearly NO:
- Human imagination is limitless
- The quality of concerts and computer games knows no bounds!

# Our best image of the future

- Continued growth...
- Rich get even richer.
- Poor will eventually also get richer but gap not eliminated.
- Much of growth in manufactured goods that use little resources. More mobiles, culture, computation, communication...
- Less transport, corals, clean water?



We need **two** sectors:  
C which grows; E (which does not)

$$W = \int_0^{\infty} e^{-\rho t} U(C, E) dt$$

The appropriate discount rate  $r$  is then

$$r = \rho + \frac{-\frac{d}{dt} U_C(C, E)}{U_C(C, E)}$$

# Relative price of "environment"

Value of environmental good is given by

$$U_E / U_C$$

The relative change in this price,  $p$ , is

$$p = \frac{\frac{d}{dt} \left( \frac{U_E}{U_C} \right)}{\left( \frac{U_E}{U_C} \right)}$$

To simplify: select utility function that combines constant elasticity of utility above with constant elasticity of substitution between E and C

$$U(C, E) = \frac{1}{1-\alpha} \left[ (1-\gamma)C^{1-\frac{1}{\sigma}} + \gamma E^{1-\frac{1}{\sigma}} \right]^{\frac{(1-\alpha)\sigma}{\sigma-1}}$$

# The relative price effect

$$p = \frac{\frac{d}{dt} \left( \frac{U_E}{U_C} \right)}{\left( \frac{U_E}{U_C} \right)} = \frac{1}{\sigma} (g_C - g_E).$$

# Formula for discounting

- not only is there a relative price effect
- but the discounting formula itself changes

# Discounting in 2 sector model

$$r = \rho + \left[ (1 - \gamma^*)\alpha + \gamma^* \frac{1}{\sigma} \right] g_C + \left[ \gamma^* \left( \alpha - \frac{1}{\sigma} \right) \right] g_E$$

Where  $\gamma^*$  is "utility share" of the environment

$$\gamma^* = \frac{\gamma E^{1-\frac{1}{\sigma}}}{(1-\gamma)C^{1-\frac{1}{\sigma}} + \gamma E^{1-\frac{1}{\sigma}}} = \frac{U_E E}{U_E E + U_C C} = \frac{\frac{U_E}{U_C} E}{\left( \frac{U_E}{U_C} E \right) + C}$$

# Comparing discount formulas

$$r = \rho + \left[ (1 - \gamma^*)\alpha + \gamma^* \frac{1}{\sigma} \right] g_C + \left[ \gamma^* \left( \alpha - \frac{1}{\sigma} \right) \right] g_E$$

$$r(t) = \rho + \alpha g_C(t)$$

# Comparison of discount rates

$$g_c = 2,5\%, \rho = 1\%, g_E = 0\%,$$

$\alpha$	$\sigma$	Convent $r$	<b>2sector</b> $R$		
0.5	0.5	2.25	<b>3.35</b>		
0.5	1	2.25	<b>2.37</b>		
0.5	1.5	2.25	<b>2.28</b>		
<b>1</b>	<b>0.5</b>	<b>3.5</b>	<b>4.24</b>		
1	1	3.5	<b>3.50</b>		
1	1.5	3.5	<b>3.44</b>		
1.5	0.5	4.75	<b>5.12</b>		
1.5	1	4.75	<b>4.62</b>		
1.5	1.5	4.75	<b>4.60</b>		



# Comparison of discount rates

$$g_c = 2,5\%, \rho = 1\%, g_E = 0\%,$$

$\alpha$	$\sigma$	Convent $r$	<b>2sector</b> $R$	<i>Price</i> $p$	TOT $R$
0.5	0.5	2.25	<b>3.35</b>	-5.00	-1.65
0.5	1	2.25	<b>2.37</b>	-2.50	-0.12
0.5	1.5	2.25	<b>2.28</b>	-1.67	0.61
<b>1</b>	<b>0.5</b>	<b>3.5</b>	<b>4.24</b>	<b>-5.00</b>	<b>-0.76</b>
1	1	3.5	<b>3.50</b>	-2.50	1.00
1	1.5	3.5	<b>3.44</b>	-1.67	1.77
1.5	0.5	4.75	<b>5.12</b>	-5.00	0.12
1.5	1	4.75	<b>4.62</b>	-2.50	2.13
1.5	1.5	4.75	<b>4.60</b>	-1.67	2.94

# Conclusions

- Relative prices CRUCIAL in long run CBA
- Discounting itself complex in 2 sector model
- Important policy conclusions for Climate
- Next step: integrated GE Climate model

# Introducing relative prices into DICE

- Stern has been criticised for low  $r$ .  $\delta=0,1$   
 $\eta=1$  and per capita  $g = 1,3$ . Total 1.4
- Nordhaus reproduced Stern-type results  
with DICE and low  $r$
- We reproduce Stern (or intermediate)  
results with Nordhaus values (high  $r$ )
- By including a small part of non-market  
sector and changing relative prices.

# An even Sterner Review

Thomas Sterner & Martin Persson

1. Comment on  $r$ ,  $\eta$  and  $\delta$
2. And on non market damages
3. Introduce Relative Prices into Debate

## 2 Changes to DICE

- The original model maximizes total discounted utility using a CRRA function
- $U(C) = C^{1-\alpha} / (1-\alpha)$
- To include the effect of changing relative prices we use a constant elasticity of substitution function of two goods:
- $U(C) = [(1-\gamma)C^{1-1/\sigma} + \gamma E^{1-1/\sigma}]^{(1-\alpha)\sigma/(\sigma-1)} / (1-\alpha)$

# Environmental Damages

- First we assume a share of environmental services in current consumption of 10%.
- We assume damage to environmental amenities will be quadratic in temperature
- At 2,5 °C damage ~ 2% current GDP
- $E(t) = E_0 / [1 + aT(t)^2]$
- *So  $E$  is actually falling due to climate ch.*
- *We assume elasticity of Substitution is .5*

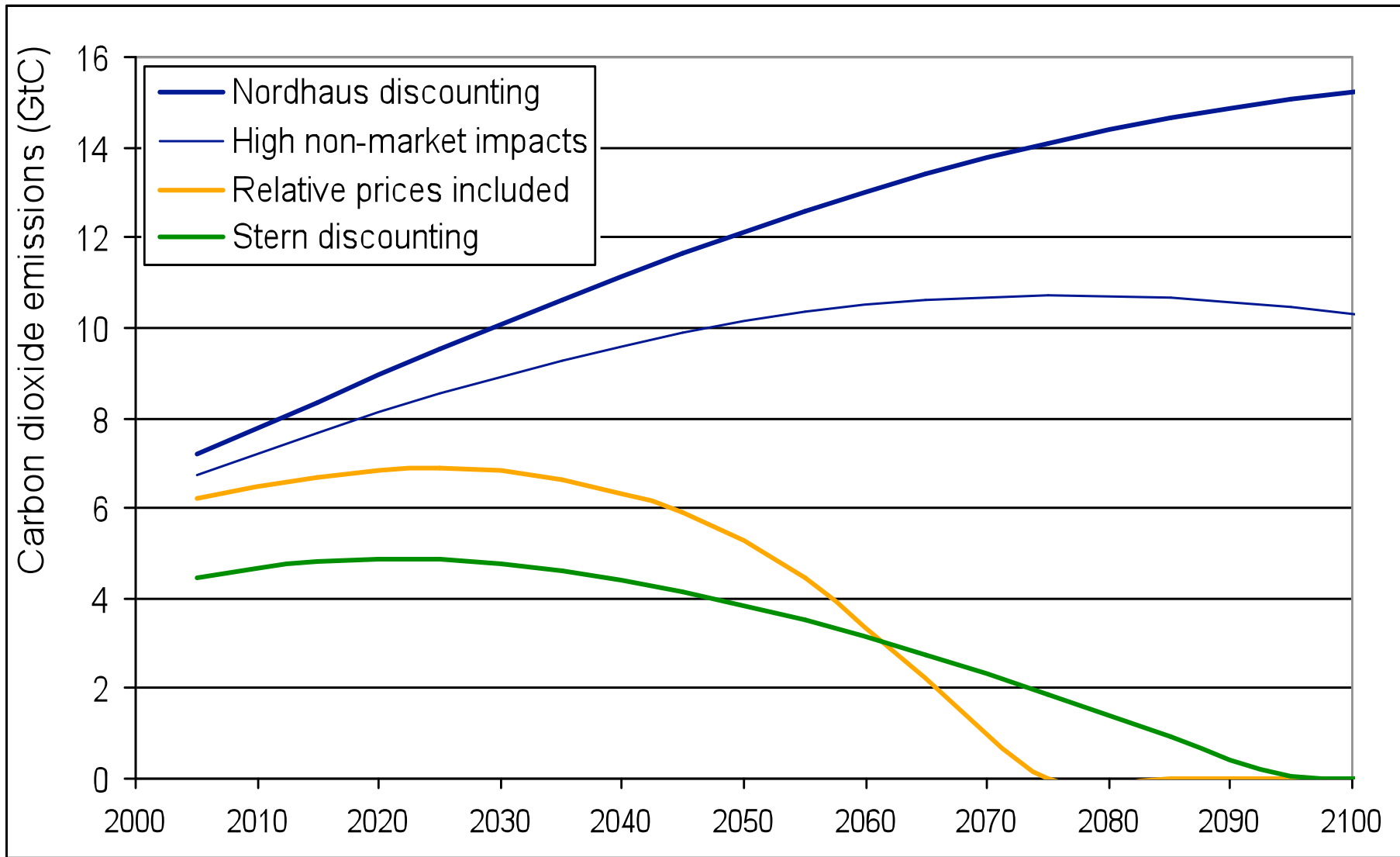


Figure 2: Optimal carbon dioxide emission paths in the DICE model for four different cases: the original model (Nordhaus discounting), the original model with high non-market impacts (High non-market impacts), the original model with low discount rate (Stern discounting) and a run where the changes in relative prices between market and non-market (environmental) goods is taken into account (Relative prices included). See text for explanation.

# Thank you very much

- More issues:
- Sensitivity,
- Relative income

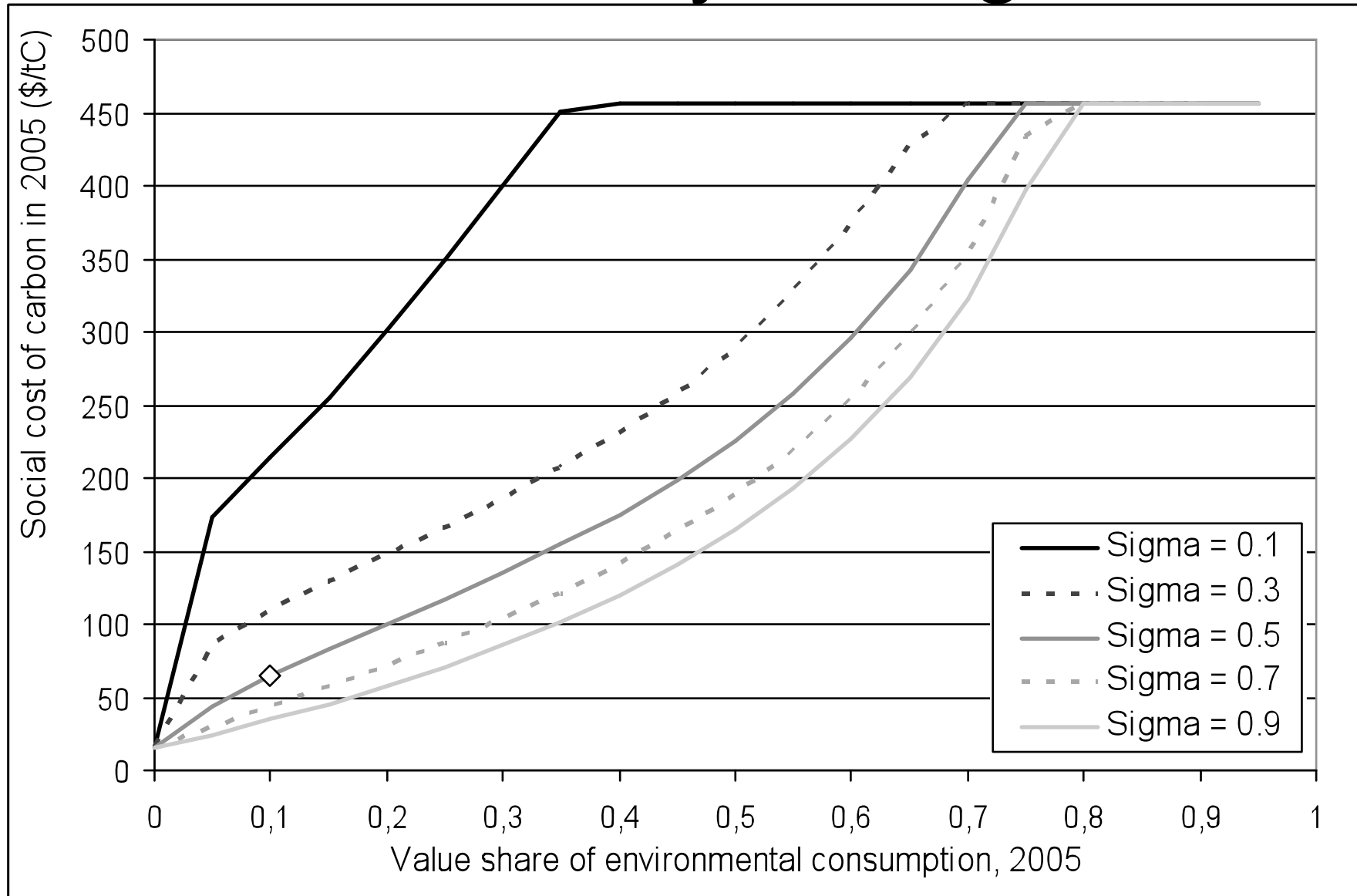


# Relative Income Hypothesis

- If  $U(c, z)$  and certainty then opt  $r$  lower (OJS, TS)
- CG OJS TS:
- If  $U(c, g_c)$  and uncertainty then optimal  $r$  is lowered because habit formation reduces the wealth effect. However habits also reduce precautionary effect (however less)
- So Net effect still  $\rightarrow$  lower discount rate



# Sensitivity testing



## Warning:

Next 10 slides: details of  $r, p$  &  $R$

- Discount rates will be the **same** if
- $\gamma^* = 0$  (Sector E plays no role for U)
- $g_C = g_E$  (Sectors E and C identical)
- $\alpha \sigma = 1$

## 2 sector discount will be lower if

- $g_C > g_E$  (Sector E grows slowly)

and

- $\alpha \sigma > 1$  (ie if substitutability is good and utility curvature very high).
- NB that normally if  $\sigma \neq 1$  and  $\alpha \sigma \neq 1$  then  $r$  in the 2 sector model will change over time

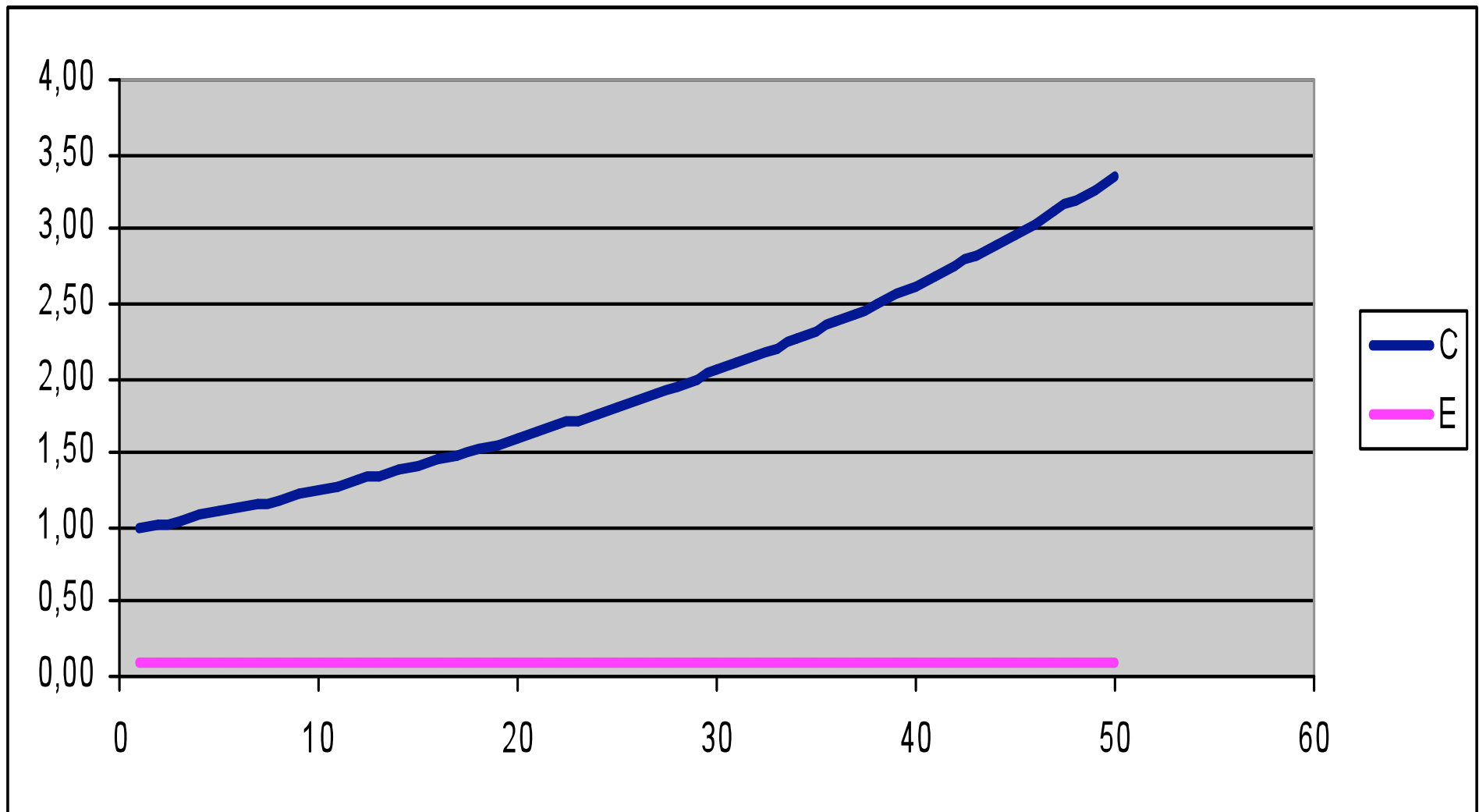
# The TOTAL discount factor

Using  $R$  to denote **the combined effect of discounting and relative price** increase of environmental goods,  
i.e.  $R=r-p$ ,

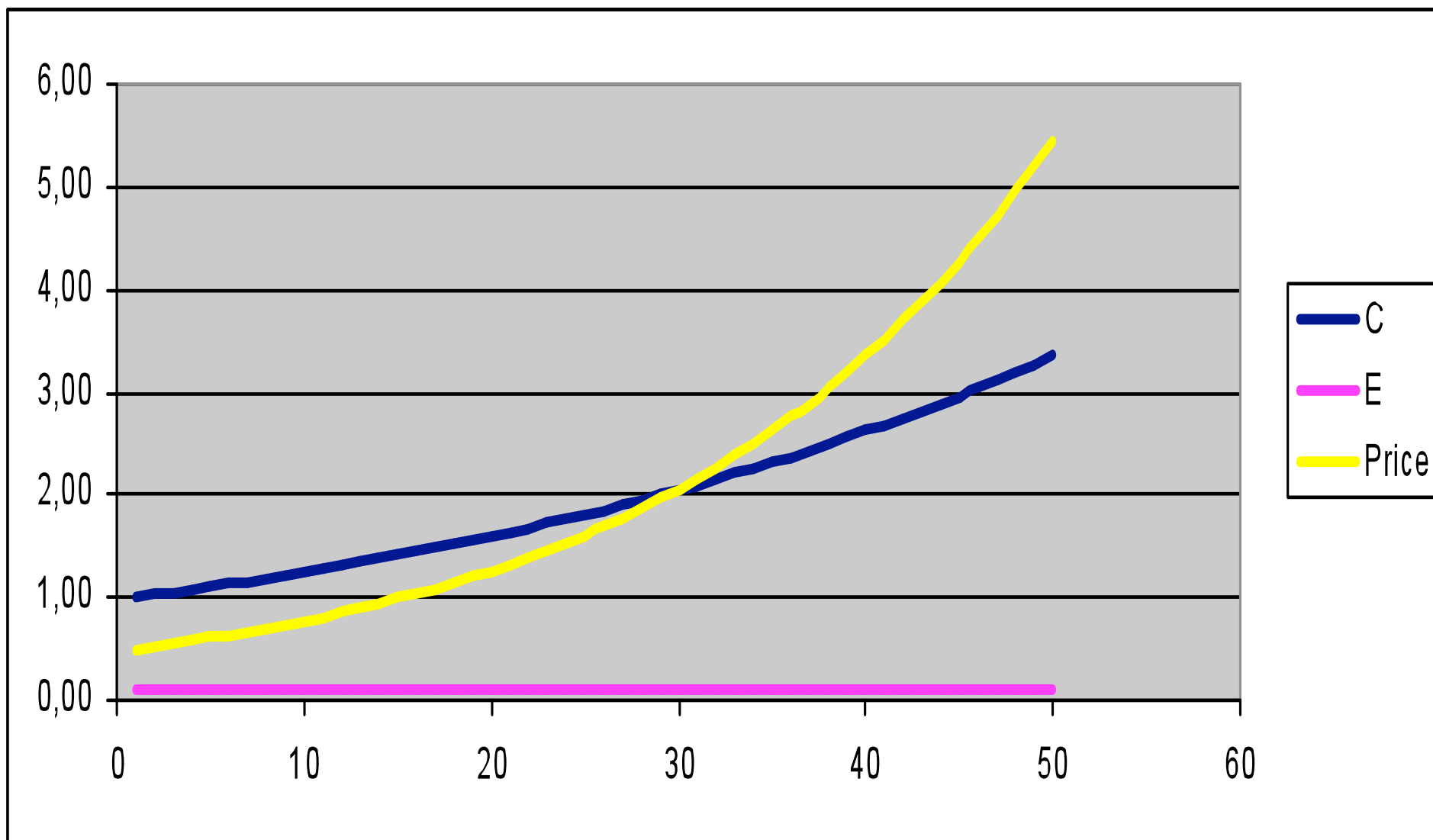
$$R = \rho + \left[ (1 - \gamma^*) \left( \alpha - \frac{1}{\sigma} \right) \right] g_C + \left[ \gamma^* \alpha + (1 - \gamma^*) \frac{1}{\sigma} \right] g_E$$

# 2 sectors, C&E with different rates

$\sigma=0,5$

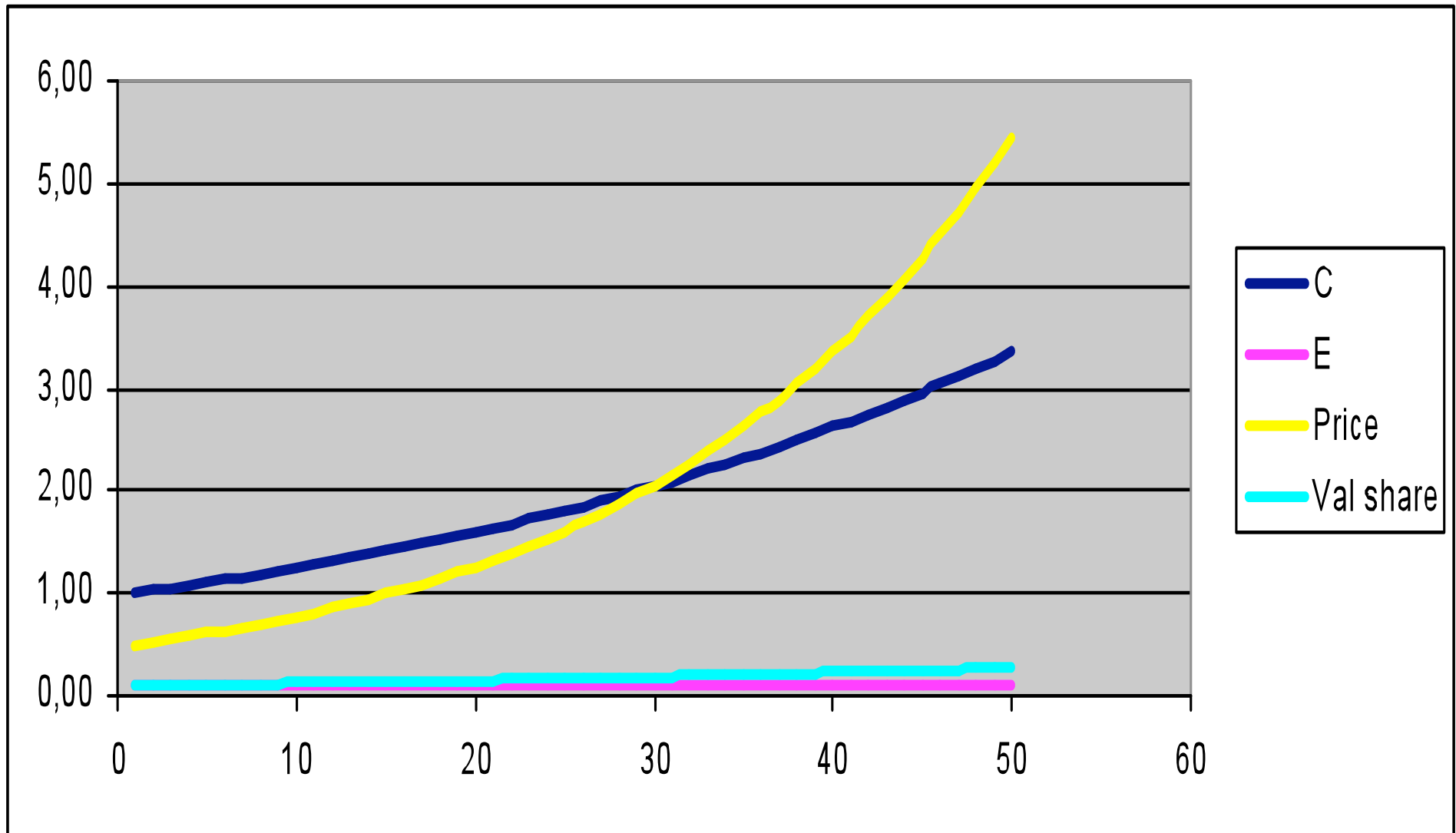


C gets bigger but the price of E  
goes up **FASTER**

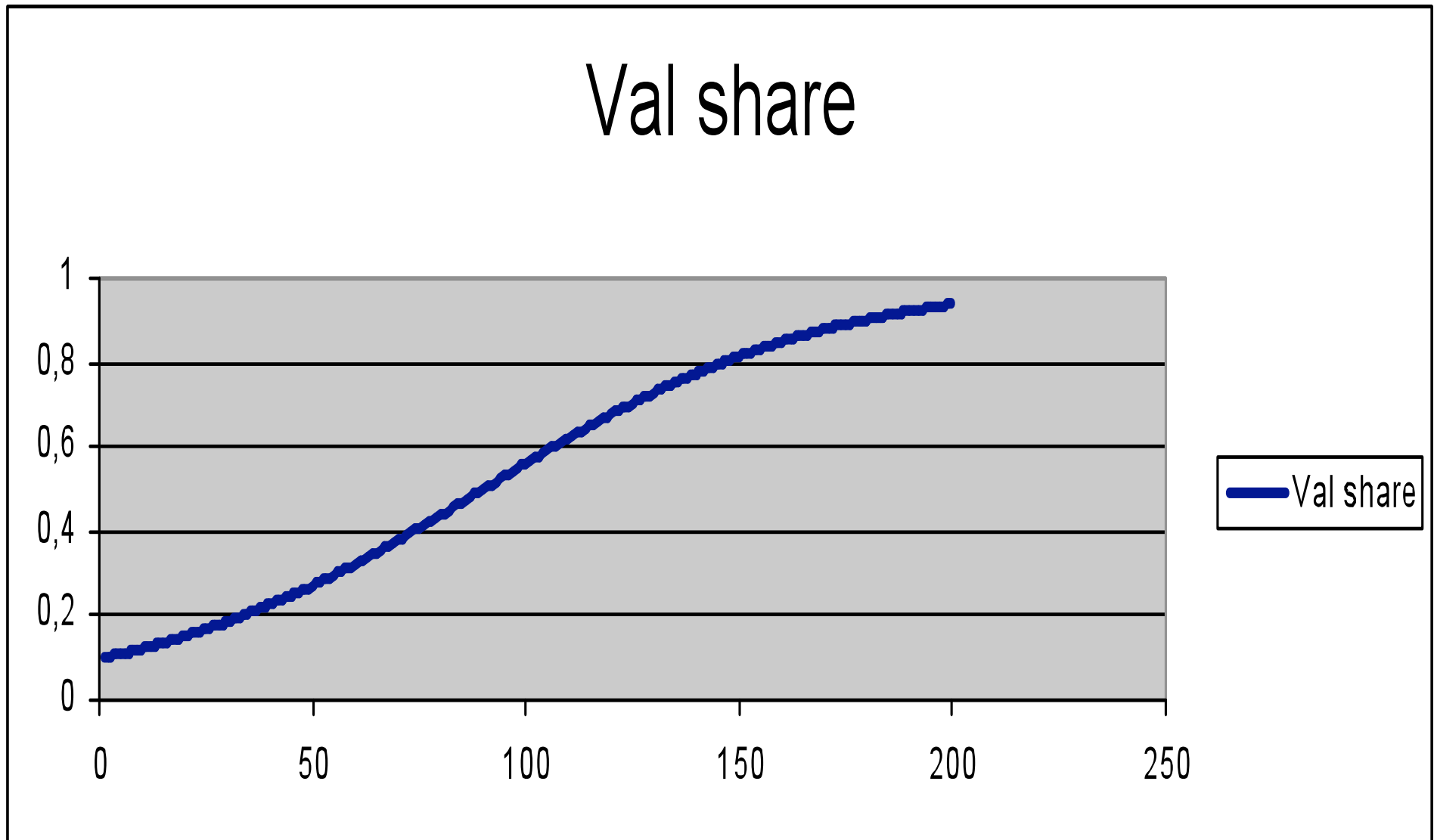




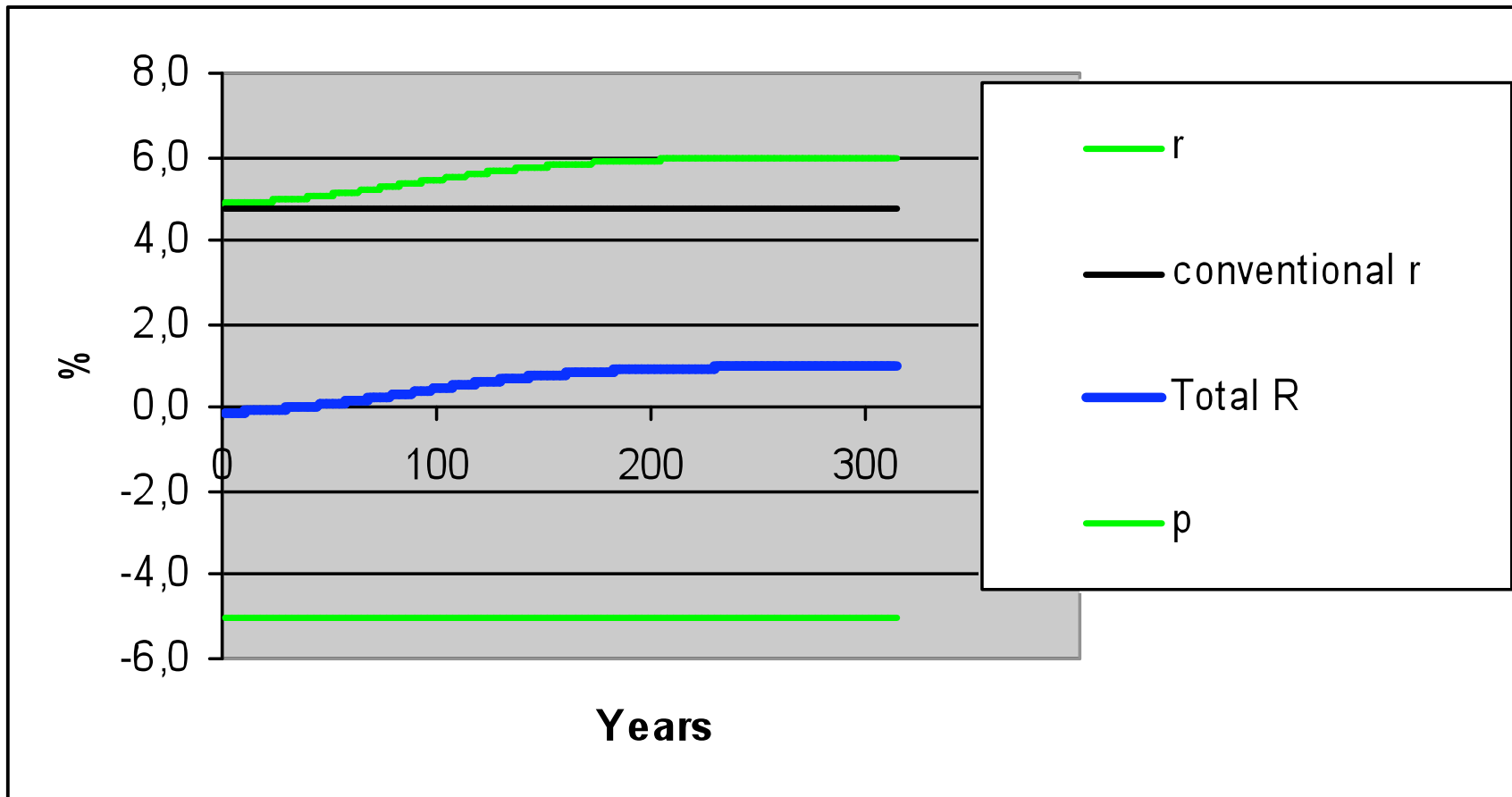
# So the value share of E rises



# After some time E dominates



Therefore variation in discount rate  
 $\rho=0.01, \sigma=0.5, \alpha=1.5, \gamma^*_0=0,1 g_C=2.5\%$



# More opinions on Climate costs

- Not reasonable to base  $r$ , in this case, on short term markets for equity or bonds
- Ethics: Reasonable to use low delta
- On top of this more non-market damages and changing relative prices!
- RISK: Uncertain outcomes with uncertain parameters in uncertain model + uncertain valuation → FAT tails
- Separate valuation of disaster risks?



# Sign of Derivatives of $r$ , $p$ , and $R$

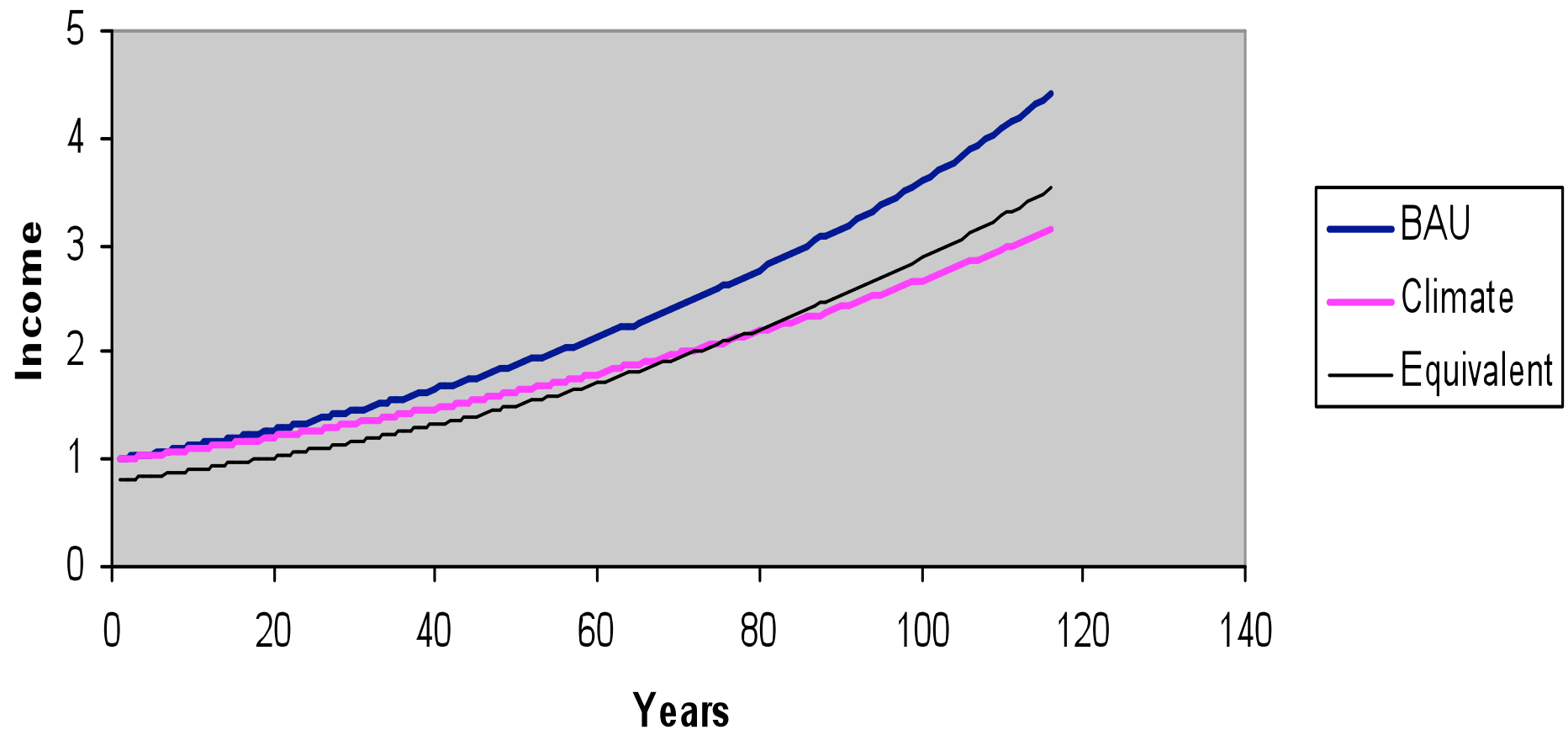
	$R$	$p$	$R = r - p$
$g_C$	+	+	- if $\alpha\sigma < 1$ + if $\alpha\sigma > 1$
$g_E$	- if $\alpha\sigma < 1$ + if $\alpha\sigma > 1$	-	+
$\alpha$	Depends on $\gamma^*$ , $g_C$ and $g_E$ (+ if $g_C > 0$ and $g_E \geq 0$ )	0	Depends on $\gamma^*$ , $g_C$ and $g_E$ (+ if $g_C > 0$ and $g_E \geq 0$ )
$\sigma$	- (if $g_C > g_E$ )	- (if $g_C > g_E$ )	+ (if $g_C > g_E$ )

# Double counting ?

- Is someone lost:
- Are we double counting when we first work out special discount formula that builds on the marginal utility of *quantities* of E and C and then also add in a relative price change?
- No: Our discount rate for the two sector model is specifically formulated in terms of rate of change of  $U_C$  !

# 5-20% For now and forever...

Presenting Future costs clearly





Discount rates will be the **same** if

- $\gamma^* = 0$  (Sector E plays no role for U)
- $g_C = g_E$  (Sectors E and C identical)
- $\alpha \sigma = 1$
- (For instance if  $\alpha = \sigma = 1$  then utility is logarithmic and substitution between E and C is good (1% change in price leads to 1% change in cons)).

# Costa & Kahn, The Rising Price of Nonmarket goods, AEA Papers &P

TABLE 1—THE VALUE OF LIFE IN 2002 DOLLARS,  
1900–2000

Year	Value of life
1900	\$427,000 (predicted)
1920	895,000 (predicted)
1940	1,377,000
1950	2,426,000
1960	2,884,000
1970	5,176,000
1980	7,393,000
2000	12,053,000 (predicted)

# More opinions on Stern & Nordhaus

- Not reasonable to base  $r$ , in this case, on short term markets for equity or bonds
- LONG run should be used. Other phenomena such as lack of aid and lack of progressive taxes
- In 1970s "everyone" recommended welfare weighting (Dasgupta, Marglin, Sen, Little & Mirrlees (1974) Drèze and Stern.  $\eta = 1$  is already quite high. Sometimes 2 was recommended but
- In practical CBA it is not used ie  $\eta=0$  !
- It would be strange to use  $\eta=0$  for all current issues and  $\eta=2$  only for decisions about the future.